# Nonlinear Analysis of a Reinforced Concrete Frame

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*Abstract:* **The recent major earthquakes have demonstrated the necessity of adequate nonlinear analysis in design of new reinforced structures. Moreover, it is not only necessary to assure buildings for non-collapse. Current studies focus also on the minimization of the economic loss. As a result, the performance based earthquake engineering (PBEE) methods are developed in order to estimate the performance levels of the structures under certain potential hazard. The overall objective of the current research is to investigate the structural behavior under seismic loading of a reinforced concrete frame. Two widely used PBEE procedures are considered - static pushover analysis and incremental dynamic analysis. In addition, the analyses are performed considering different variable loading. At the end, a comparison of the two methods is made in order to outline the advantages of both methods and to estimate their applicability in the contemporary structural design.**

*Keywords:* **nonlinear analysis, static pushover analysis, incremental dynamic analysis, reinforced concrete frame.**

## I.   INTRODUCTION

The existing buildings in Bulgaria are mainly reinforced concrete structures. These structures show very complex behavior due to the consequence of the properties of constituent materials and to their work in conjunction. The behavior differs significantly from the classical linear behavior elastic one, even for low levels of loading. Therefore, the precise seismic design is crucial for the life of the structures.

The recent major earthquake (Northridge 1994, Kobe 1995, Izmir 1999, Sichuan 2008) that occurred have witnessed that, although the structures meet the requirement of non-collapse, the economic losses are incredibly high due to inadequate understanding of the structural performance especially under failure loads. As a result, the performance-based earthquake engineering (PBEE) methods have been developed as an alternative to the current based practices. The purpose of PBEE procedures to estimate performance levels of structures under certain potential hazard, considering the uncertainties both in the analysis of potential hazard as well as in the assessment of the structural response.

Incremental dynamic analysis (IDA) and modal pushover analysis are probably the most significant methods in this aspect. Modal pushover analysis, despite its approximate nature, is widely used as the simplified nonlinear static method to evaluate seismic demands of structures. Its main purpose is to assess the structural performance by estimating the strength and deformation capacities using static, nonlinear analysis and comparing these capacities with the demands at the corresponding performance levels. The assessment is based on the estimation of important structural parameters, such as global and inter-story drift, element deformations and internal forces. The analysis accounts for the geometrical nonlinearity and material inelasticity, as well as the redistribution of internal forces.

The incremental dynamic analysis is proposed by Bertero in the late seventies and since then this method is highly developed. Vamvatsikos and Cornell have great contribution on this aspect by introducing a computer algorithm for implementing incremental dynamic analysis. IDA was identified as the state-of-the-art method to assess the performance level of structures and it is adopted by Federal Emergency Managment Agency. Despite the fact that it is a computational demanding technique, the use of IDA allows to estimate the safety of the structure subjected to a set of various earthquake as well as to ensure that the structure meets a designated level of serviceability.

## II. STATE-OF-THE-ART

### A. Nonlinear Static Procedure:

The nonlinear static procedure or pushover analysis is an approximate analysis method for estimating seismic deformation demands in structures as well as their local and global capacities. As such, it is widely used by practicing engineers for the evaluation of the safety of structures subjected to earthquake excitations. However this procedure has its limitations - the use of pushover analysis is recommended for structures in which higher mode effects are insignificant. Otherwise a linear dynamic analysis should be performed along with the pushover analysis.

The pushover analysis is firstly introduced by Gulkan and Sozen (1974) and Saiidi and Sozen (1981). Saiidi and Sozen suggested a model that uses the moment-curvature relationships of the members to derive the top-level displacement vs. shape moment curve of the multi-degree-of-freedom (MDOF) system. To compute the force displacement characteristics of a single-degree-of-freedom (SDOF) system, the curve is presented as bilinear. The deflected shape at yielding is assumed as characteristic vibration shape of the structure. This procedure is subsequently developed by different researchers.

Lawson, Vance and Krawinkler (1994) examine the area of applicability and highlight the encountered difficulties. Four multistory steel structures varying in height are analyzed under different patterns of static loading (uniform, triangular and modal using the SRSS method to combine the modal shapes and their spectral amplifications). The results were compared to the results obtained from the dynamic analysis. The correlation between static and dynamic responses is found sufficient for the low-rise buildings. Nevertheless, the results for the high-rise structures are found inadequate due to the high mode effects.

Faella (1996) compares the response of three, six and nine story buildings subjected to artificial and real earthquakes by means of pushover analysis. He makes the conclusion that nonlinear static analysis can identify collapse mechanisms and critical regions. The report shows that the sum of the maximum dynamic drifts is larger than the maximum dynamic roof displacement, due to the random nature of earthquake loading. Thus, it is suggested to compute the static inter-story displacement to obtain the maximum roof displacement. The author also discusses the difficulties with static-dynamic comparison in case of a strong-motion input rich in long period frequencies.

In his research, Krawinkler (1995) explains the theoretical limitations of nonlinear static analysis method as well as the procedures for the estimation of the target displacement. It is concluded that pushover analysis cannot disclose performance problems caused by changes in the inelastic dynamic characteristics, due to higher mode effects. Later, Krawinkler and Seneviratna (1998) summarize the basic concepts of the method. Their study outline the main characteristics that can be defined by applying of pushover analysis:

- Accurate force demands on potentially brittle elements, such as axial demands on columns, moment demands on beam-to-column connections or shear forces demands on short, shear dominated elements.

- Estimations of the deformation demands on elements that have to deform inelastically, in order to dissipate energy.

- Significance of the strength deterioration of different members on the overall structural stability.

- Identification of the critical regions, where the inelastic deformations are expected to be high.

- Identification of strength irregularities in plan or elevation that cause changes in the dynamic characteristics in the inelastic range.

- Estimates of the interstory drifts, accounting for strength and stiffness discontinuities to control the damage on non-structural elements.

- Sequence of the member's yielding and failure and the progress of the overall capacity curve of the structure.

- Verification of the adequacy of the load path, considering all the elements of the system, both structural and non-structural.

### B. Incremental Dynamic Analysis (IDA):

Incremental Dynamic Analysis (IDA) estimates the overall behavior of structures, from their elastic response through yielding and nonlinear response and all the way to global dynamic instability. The analysis involves performing a series of nonlinear dynamic analyses of a certain structure subjected to a suite of ground motions of varying intensities. The

method also includes plotting a measure of the ground motion intensity against a response parameter (demand measure). As the curve in this plot becomes flat, the global collapse capacity is reached. At this point a small increase in the ground motion intensity evokes a large increase in the structural response (Villaverde 2007).

The idea of the incremental dynamic analyses is introduced by Bertero. However, the development of this method has begun in the recent years. Vamvatsikos and Cornell (2002) describe the incremental dynamic analysis method in detail. In their study, they define intensity-response curves for several structures, examine the properties of the curves and propose techniques to perform an incremental dynamic analysis. They observe that incremental dynamic analyses simultaneously asses the seismic demands on structures and their global capacities. The response-intensity curves show some unusual properties of such as non-monotonic behavior, discontinuities, multiple collapse capacities, and their extreme variability from ground motion to ground motion.

The research summarizes the general properties of the incremental dynamic analysis:

- The incremental dynamic analysis can be performed for a single scaled ground motion or for a multiple record of scaled ground motions.

- The slope of the IDA curves is a significant indicator of the structural response. Furthermore, all curves display a noticeable region of elastic response. In this region damage measure is proportional to intensity measure.

- To define the capacity of the structure under incremental dynamic analysis two characteristics can be used – damage measures (DM) and intensity measures (IM). The damage measure based rule is generated from the assumption that if damage measure exceeds a certain value, the limit state will be exceeded. This method gives an accurate prediction for the performance levels other than collapse.

- In case of collapse capacity, as DM-based rule requires great computational power, an IM-based rule is used. In IM-based rule, the IDA curves are divided in region of non-collapse and region of collapse.

- Due to structural resurrection (the phenomena of decreasing the structural damage while increasing the intensity of ground motion) there are several points on the IDA curves satisfying the limit state. Therefore, for a DM-based rule, the lowest point is conservatively used as the limit state point. As for the IM-based rule, the last point of the curve with slope of 20% of the elastic slope is considered as a capacity point.

- An important characteristic of the multi-record IDA curve is the "dispersion" of the responses in the inelastic range. This phenomenon represents the randomness of the different earthquakes.

Consequently, Vamvatsikos and Cornell develop a procedure for application of the IDA procedure. In their study of 2004, they explore the behavior of a 9-story steel moment-resisting frame subjected to seismic actions. The purpose of the study is to show how to apply incremental dynamic analysis efficiently, how to interpret the results, and how to apply these results to performance-based earthquake engineering (Vamvatsikos and Cornell 2004).

## III. NUMERICAL TESTING

### A. *Structural Properties and Loading:*

The construction is 3-story one-bay reinforced concrete structure for administrative purposes. The height of each floor is 3.6m, the total height is 10,8m. The width of the bay is 7,2m. The distance between frames is 6m.
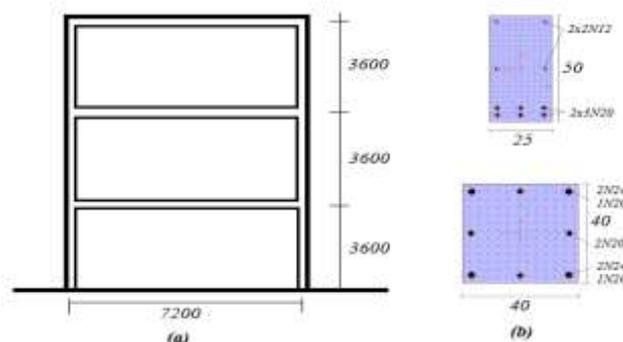


**Fig. 1: Frame Properties: (a) Evaluation of the Building (b) Frame Sections – Frame and Column**

The building is primary designed as administrative building or building category B according p.6.3.1 of Eurocode EN-1991-1-1 (CEN 2001). It is considered that after construction of the building the category is changed to category A, C5, or D and respectively the variable loading has changed.

**Table I: Categories of Use According To Euro code En-1991-1-1**

| Category | Specific Use |
|---|---|
| A | Areas for domestic and residential activities |
| B | Office areas |
| C5 | Areas susceptible to large crowds, e.g. in buildings for public events |
| D | Shopping areas |

**Table II: Variable Loading**

| | Cat. B | Cat. A | Cat. C5 | Cat. D |
|---|---|---|---|---|
| $q_k[kN/m^2]$ | 3 | 2 | 7.5 | 5 |

**Modeling approach:**

The building models are developed in OpenSees, which stands for Open System for Earthquake Engineering Simulation. The sections are modeled using the fiber element approach. The approach consists of the division of each material that comprises the section in multiple fibers. Each fiber is assigned to a material represented by its stress-strain relationship. The elements are modeled using distributed nonlinearity approach. In these models the plasticity is distributed along the element and this gives more accurate description of the nonlinear behavior of the model.
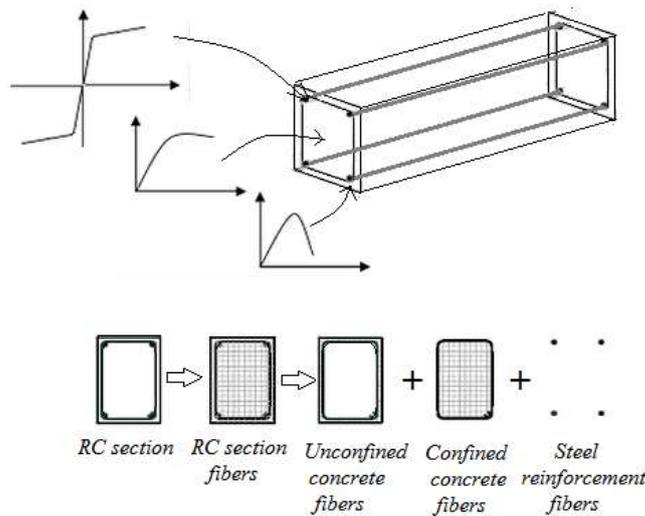


**Fig. 2: Fiber Modeling Approach**

**Material Properties:**

The materials used for the construction are concrete C30/37 and the steel S275. The computational model of the concrete and steel are respectively Concrete 01 and Steel01.

*Concrete:*

The confined and unconfined concrete in the fiber sections are modeled using Kent-Scott-Park concrete model with degradating unloading/reloading stiffness. This model, named Concrete01 in OpenSees, assumes no tensile strength for concrete. The concrete 28 day compressive strength (fpc), concrete strain at maximum strength (epsc0), and concrete crushing strain (epsU) and strength (fpcu) are required input for this concrete. These parameters are obtained by Eurocode 2, Section 3 (CEN 2002) The initial slope for the model is given by 2*fpc/epsc0. The concrete strength remains constant beyond the crushing point. The crushing strength for the unconfined (cover) concrete is considered as 0 as shown in.

Page | 159

**Table III: Concrete Properties**

|  | $fpc(kN)$ | $epsc0$ | $E0(kN)$ | $fpcu$ | $epsU$ |
|---|---|---|---|---|---|
| *Cover Concrete C30/37* | 38 | 0.0022 | 33000 | 0 | 0.02 |
| *Core Concrete C30/37* | 38 | 0.0022 | 33000 | 30 | 0.02 |

*Steel:*

In this research a bilinear steel material with kinematic hardening and optional isotropic hardening described by a non-linear evolution equation, named Steel01 in Opensees, is used to represent the stress-strain response of the main longitudinal steel bars. This model is characterized by an initial elastic modulus (E0), a yield stress (Fy) and a post-yield modulus defined by the strain hardening parameter (b). It has an elastic phase before yielding and a strain-hardening part after yielding. Isotropic hardening is not considered; therefore the isotopic hardening parameters are left default.

**Table IV: Steel Properties**

|  | $Fy(kN)$ | $E0$ | $b$ | $a1$ | $a2$ | $a3$ | $a4$ |
|---|---|---|---|---|---|---|---|
| *Steel S275* | 40 | 30000 | 0.02 | 1 | 0 | 1 | 0 |

*B. Static Pushover Analysis:*

Static Pushover Analysis is based on the equivalent lateral force method. To perform such an analysis some simplification assumptions are made:

- The structure is idealized as system with lumped masses and one translational degree of freedom per floor along each of two orthogonal horizontal directions. Masses are derived from weights by dividing the weight by the acceleration due to gravity.

- The building's floors are infinitely rigid and therefore undeformable in their own plane.

- A building is analyzed along an in-plane horizontal component of ground motion.

- The direct outcome of the procedure is normal lateral forces in the direction of the ground motion being considered. According to Eurocode EN 1998-1, Annex B, the normalized lateral forces are defined as:

$$\bar{F_\iota} = m_i \Phi_i$$

*Where $m_i$ is the mass of the i-th floor and $\Phi_i$ is the vector of the horizontal displacement at i-th floor*

The analysis is run in two steps. In stage one the building is loaded only with vertical loading. The loading combination includes self-weight, permanent loading and variable loadings (with partial factor $\psi_2$=0,3). Forces control is applied. In the second stage the normalized lateral forces are concentrated in the nodes and displacement control is applied.

*C. Incremental Dynamic Analysis:*

**Design response spectrum:**

NEHRP (2003) provides information about maximum considered earthquakes with uniform likelihood of occurrence of 2% in 50 years, with returning period 2500 years. The damping of the structures is considered 5%. The spectral acceleration is divided into two groups: for short period (0.2sec) and for 1-second period, the equations are presented hereby:

$$S_{DS} = \frac{2}{3} F_a S_s$$

$$S_{D1} = \frac{2}{3} F_v S_1$$

*Where $F_a$ is the coefficient for site class (short period), $F_v$ is the coefficient for site class (1-second period), $S_s$ is the mapped spectral acceleration at short period, $S_1$ is the mapped spectral acceleration at 1-second period*

The building is considered to be located in site category B according to NEHRP:

Page | 160

**Table II: P Arameters Used In Design Spectral Acceleration Curve**

| Site | $S_s$ | $S_1$ | $F_a$ | $F_v$ | $S_{DS}$ | $S_{D1}$ |
|---|---|---|---|---|---|---|
| Site B | 2.1 | 0.93 | 1.00 | 1.00 | 1.4 | 0.62 |

**Natural Damping**

In case of inelastic analysis the damping should be considered between 5% to 20% (Chopra 2007). In this study a 5% natural damping has been used.

**Fundamental Period**

The fundamental period of vibration is one of the most important pieces of information required for dynamic analysis. In this research it is obtain by means of Eigen analysis.

**Table VI: Fundamental Periods**

| | Cat. B | Cat. A | Cat. C5 | Cat. D |
|---|---|---|---|---|
| Period (s) | 0.3855 | 0.3813 | 0.4336 | 0.41 |

**Selection of ground motion**

In the current study the Northridge earthquake is used for the analysis

**Table VII: North Ridge Earthquake Parameters**

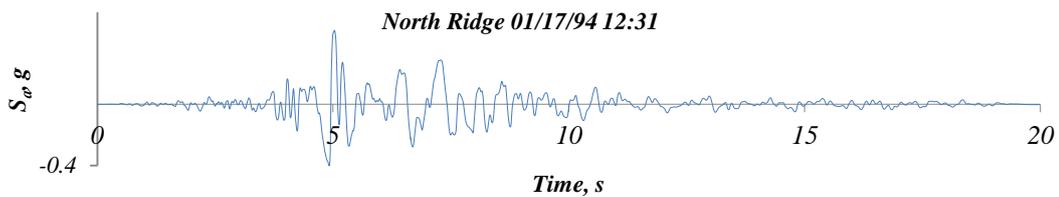| Earthquake | Station | Direction | PGA |
|---|---|---|---|
| North Ridge 01/17/94 12:31 | Canyon Country | 270 | 0.410 |



**Fig. 3: Record of North Ridge 01/17/94 12:31**

To perform the incremental dynamic analysis the earthquake record is scaled to assure that pseudo accelerations are equal to the design spectral accelerations. The scale factors are obtained in relation with the fundamental period.

**Table VIII: Scaling Factors, North Ridge 01/17/94 12:31, PGA 0.482g**

| | Cat. B | Cat. A | Cat. C5 | Cat. D |
|---|---|---|---|---|
| Scaling factor | 1.388 | 1.378 | 1.306 | 1.356 |
| Scaled PGA (g) | 0.669 | 0.664 | 0.629 | 0.654 |

*D. Results*

The incremental dynamic analysis is based on single record IDA. In each IDA curve only one parameter is varied in time – the variable loading. In the present incremental analysis the interstory drift is used as damaged measure. The intensity measure is the first-mode spectral acceleration $S_a(T_1, 5\%)$.Comparison is shown in the tables below:

**Table IX: Analyses Comparison - Target Displacement**

| | Cat. B | Cat. A | Cat. C5 | Cat. D |
|---|---|---|---|---|
| Target displacement (IDA, scaled ground motion) | 4.652 | 4.419 | 5.37 | 5.202 |
| Target displacement (Pushover Analysis) | 7.40 | 7.55 | 5.73 | 6.1 |
| Difference (%) | 37 | 41 | 6 | 14 |

**Table X: Analyses Comparison – Spectral Acceleration**

|  | *Cat. B* | *Cat. A* | *Cat. C5* | *Cat. D* |
|---|---|---|---|---|
| *Sa, g (IDA, scaled ground motion)* | 1.07 | 1.1 | 0.81 | 0.91 |
| *Sa,g (Pushover Analysis)* | 0.529 | 0.546 | 0.397 | 0.454 |
| *Difference (%)* | 50 | 50 | 50 | 50 |

The comparison between the static pushover analysis and the incremental dynamic analysis is represented graphically on the pictures below:
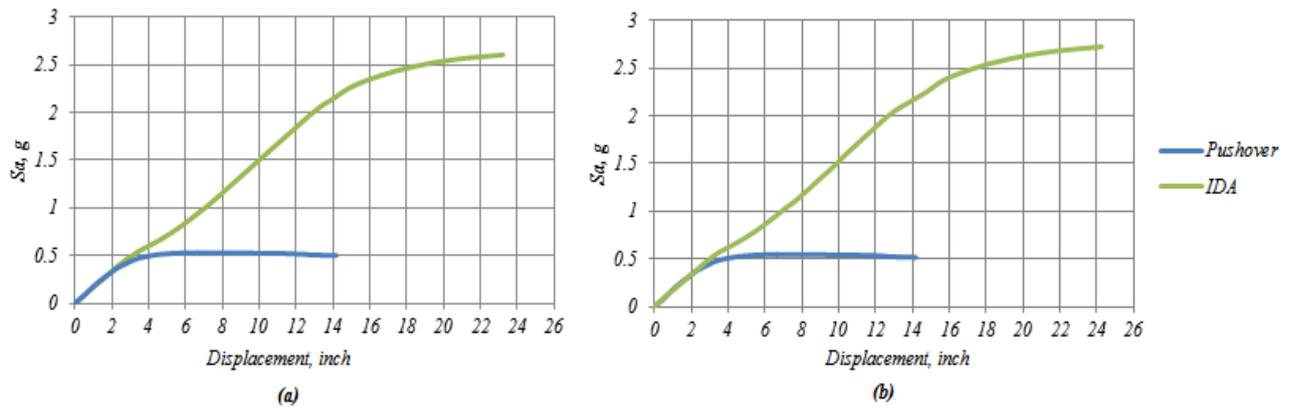


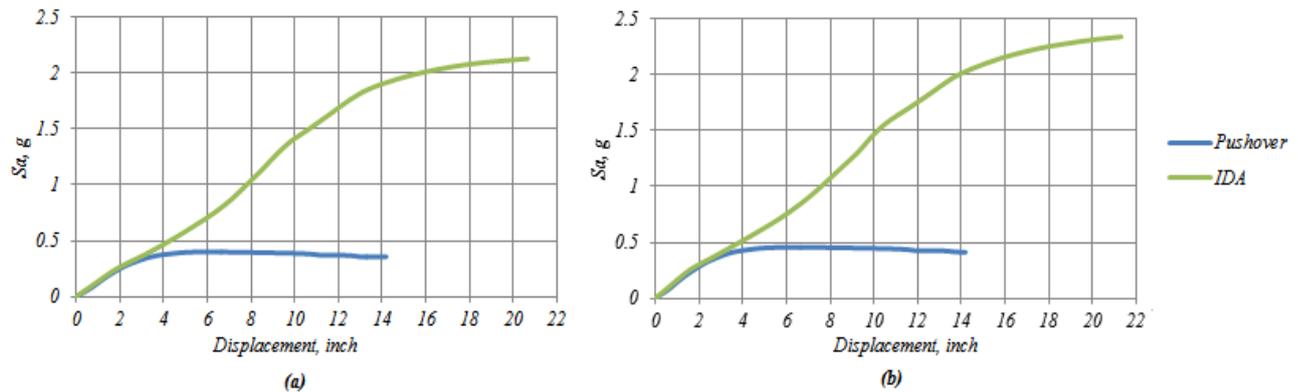**Fig. 4: Nonlinear Analyses Comparison: (a) Cat. B and (b)Cat.A**



**Fig. 5: Nonlinear Analyses Comparison: (a) Cat. C5 and (b)Cat.D**

## IV.  CONCLUSION

The pushover analysis appears to give accurate results in the elastic stage. The results are comparable to those obtained by incremental dynamic analysis. However, in the nonlinear stage, pushover analysis tends to be very conservative in comparison with the IDA. By comparing the intensity measures of the analyses, it is visible that the target displacement in static pushover analysis is obtained when the building is subjected to a loading which is two times smaller than the one predicted by incremental dynamic analysis. In addition, the difference in the intensity measures is not influenced by the variable loading. On the other hand, if we compare the damage measure, the difference of the target displacements obtained from different methods is highly influenced by the intensity of the loading.

In the region of first yielding in pushover curve, the IDA curve remains approximately linear and follows the elastic slope.

However, in this region some softening is observed and the softening appears to be bigger in the more heavily loaded structures.

The final conclusion that can be made is that when the pushover curve is in its negative slope the IDA curve shows signs of bigger and constant softening which eventually will lead to collapse. This tendency can be observed in all the cases with no regard to the loading.

The present research has some limitations to be investigated in future works. Firstly, the research focuses on low-rise frames. A study on high rise buildings can significantly contribute to the topic. Secondly, for incremental dynamic analysis a greater number of ground motions would increase the confidence of the observed trend. Therefore it is highly recommended this study to be extended with more ground motions, however, this will demand more computational power.

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